

11.1 Dilations

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Resource Locker

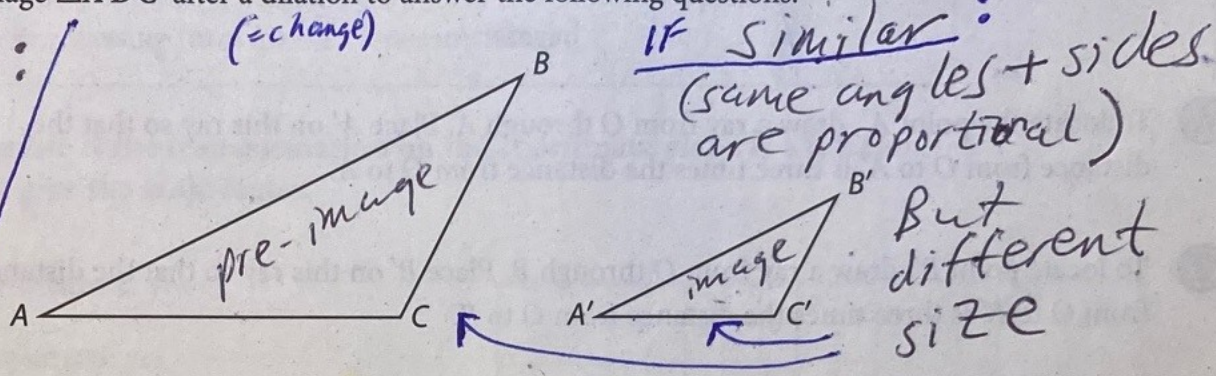
Essential Question: How does a dilation transform a figure?

Explore 1 Investigating Properties of Dilations

A **dilation** is a transformation that can change the size of a polygon but leaves the shape unchanged. A dilation has a *center of dilation* and a *scale factor* which together determine the position and size of the image of a figure after the dilation.

Use $\triangle ABC$ and its image $\triangle A'B'C'$ after a dilation to answer the following questions.

IF Congruent:
EXACTLY THE
SAME SIZE



A Use a ruler to measure the following lengths. Measure to the nearest tenth of a centimeter.

$AB = 6$ cm	$A'B' = 3$ cm
$AC = 4$ cm	$A'C' = 2$ cm
$BC = 3$ cm	$B'C' = 1.5$ cm

B Use a protractor to measure the corresponding angles.

$m\angle A = 22$	$m\angle A' = 22$
$m\angle B = 33$	$m\angle B' = 33$
$m\angle C = 125$	$m\angle C' = 125$

C Complete the following ratios

$k = \frac{A'B'}{AB} = \frac{3}{6} = \frac{1}{2}$ $\frac{A'C'}{AC} = \frac{2}{4} = \frac{1}{2}$ $\frac{B'C'}{BC} = \frac{1.5}{3} = \frac{1}{2}$

image / pre-image = k = scale factor

so $k = \frac{1}{2}$
scale factor

Reflect

1. What do you notice about the corresponding sides of the figures? What do you notice about the corresponding angles?

The ratios of the lengths of corresponding sides are equal ($\frac{1}{2}$). Corresponding angles are congruent.

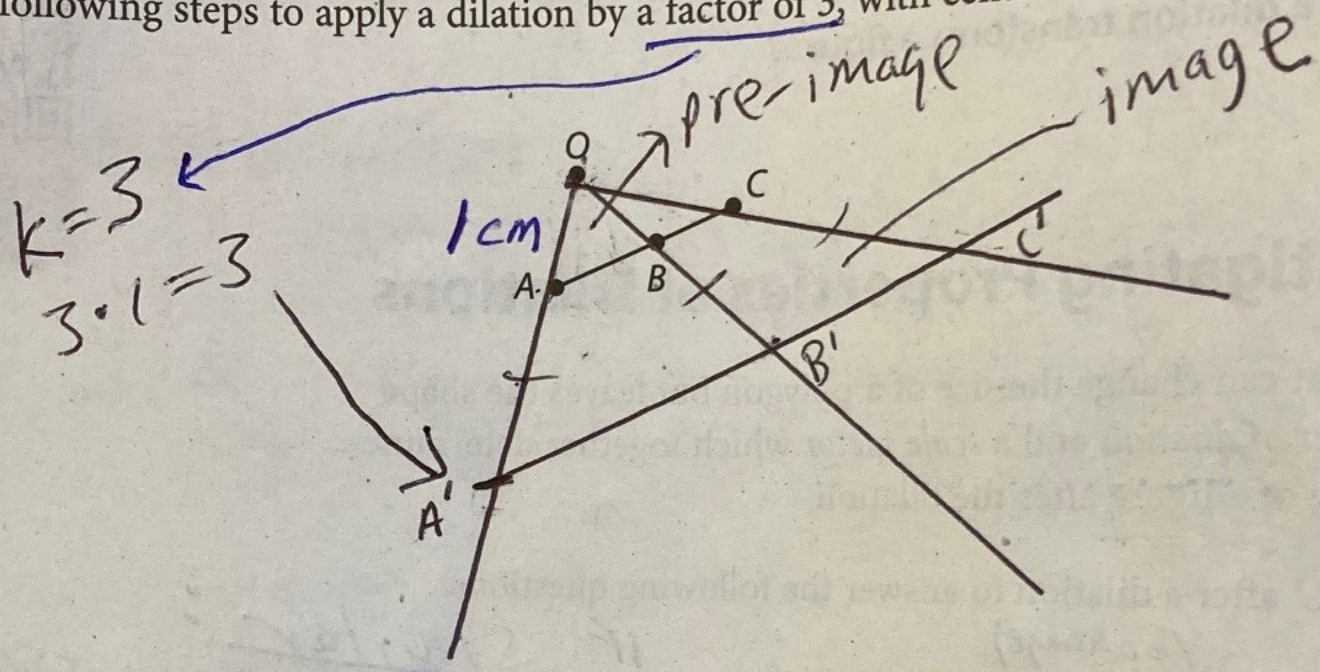
2. **Discussion** What similarities are there between reflections, translations, rotations, and dilations? What is the difference?

Similarities: All of them preserve angle measures
Difference: Dilations change side lengths.

Explore 2 Dilating a Line Segment

The dilation of a line segment (the pre-image) is a line segment whose length is the product of the scale factor and the length of the pre-image.

Use the following steps to apply a dilation by a factor of 3, with center at the point O, to \overline{AC} .



- (A) To locate the point A' , draw a ray from O through A . Place A' on this ray so that the distance from O to A' is three times the distance from O to A .
- (B) To locate point B' , draw a ray from O through B . Place B' on this ray so that the distance from O to B' is three times the distance from O to B .
- (C) To locate point C' , draw a ray from O through C . Place C' on this ray so that the distance from O to C' is three times the distance from O to C .
- (D) Draw a line through A' , B' , and C' .
- (E) Measure \overline{AB} , \overline{AC} , and \overline{BC} . Measure $\overline{A'B'}$, $\overline{A'C'}$, and $\overline{B'C'}$. Make a conjecture about the lengths of segments that have been dilated.

Reflect

- 3. Make a conjecture about the length of the image of a 4 cm segment after a dilation with scale factor k . Can the image ever be shorter than the preimage?

Think projector (p. 579)

Explain 1 Applying Properties of Dilations

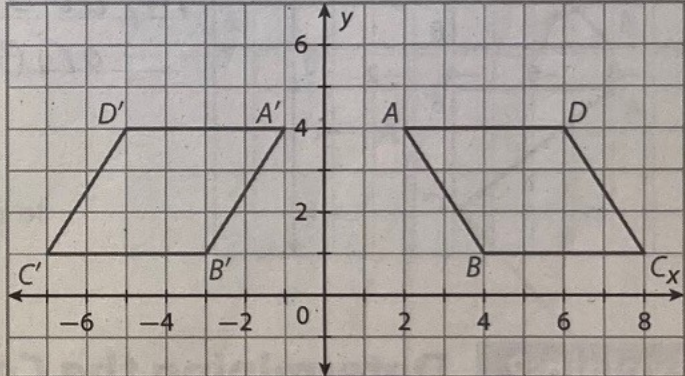
The **center of dilation** is the fixed point about which all other points are transformed by a dilation. The ratio of the lengths of corresponding sides in the image and the preimage is called the **scale factor**. $= k$

Properties of Dilations

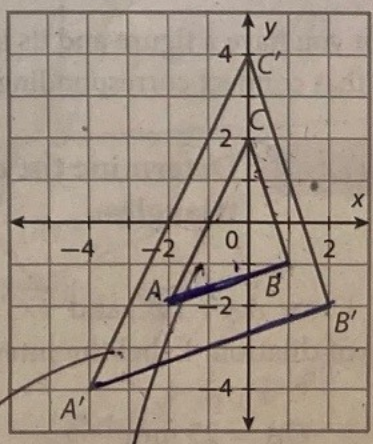
- Dilations preserve angle measure. ✓
- Dilations preserve betweenness. ✓
- Dilations preserve collinearity. ✓
- Dilations preserve orientation. ✓ *pre-image* → *image* YES NO
- Dilations map a line segment (the pre-image) to another line segment whose image length is the product of the scale factor and the length of the pre-image. ✓
- Dilations map a line not passing through the center of dilation to a parallel line and leave a line passing through the center unchanged. ✓

Example 1 Determine if the transformation on the coordinate plane is a dilation. If it is, give the scale factor.

- A** Preserves angle measure: yes
 Preserves betweenness: yes
 Preserves collinearity: yes
 Preserves orientation: no
 Ratio of corresponding sides: 1 : 1
 Is this transformation a dilation? No, it does not preserve orientation.



- B** Preserves angle measure (Y/N) Y
 Preserves betweenness (Y/N) Y
 Preserves collinearity (Y/N) Y
 Preserves orientation (Y/N) Y
 Scale Factor 2 or 2/1 or 2 to 1
 Is this transformation a dilation? YES



$k = \frac{\text{image}}{\text{pre-image}} = \frac{6}{3} = 2$
 and $\frac{8}{4} = 2$ and $\frac{6}{3} = 2$
 = count squares or measure

pre-image
 image

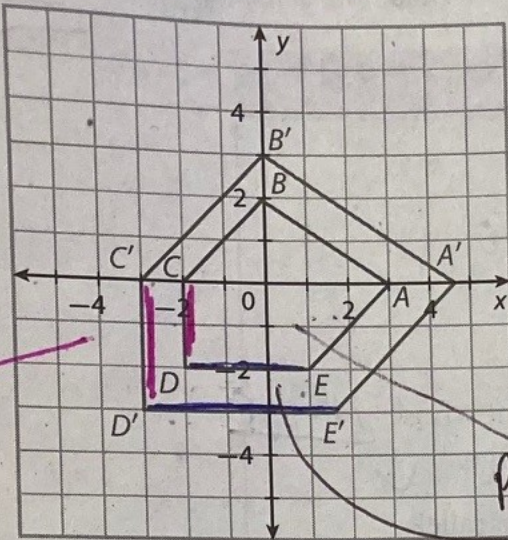
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Your Turn

Determine if the transformations are dilations.

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5.



Yes, all properties are true.

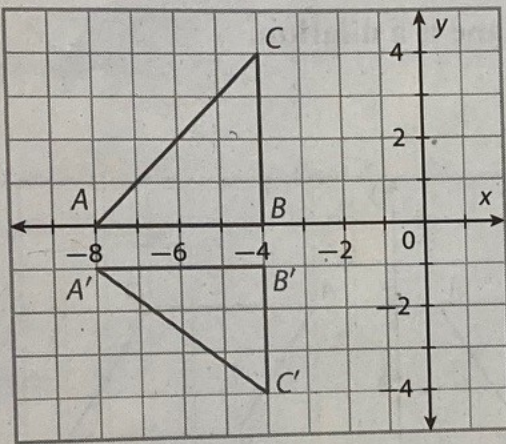
$k = 3/2$

pre-image

~~image~~

$$k = \frac{\text{image}}{\text{pre-image}} = \frac{4.5}{3} = \frac{3}{2} \text{ or } 1.5$$

6.



No, does not preserve orientation.

Explain 2 Determining the Center and Scale of a Dilation

When you have a figure and its image after dilation, you can find the center of dilation by drawing lines that connect corresponding vertices. These lines will intersect at the center of dilation.

Example 2 Determine the center of dilation and the scale factor of the dilation of the triangles.

A Draw $\overrightarrow{AA'}$, $\overrightarrow{BB'}$, and $\overrightarrow{CC'}$. The point where the lines cross is the center of dilation. Label the intersection O. Measure to find the scale factor.

OA = 25 mm

OB = 13 mm

OC = 19 mm

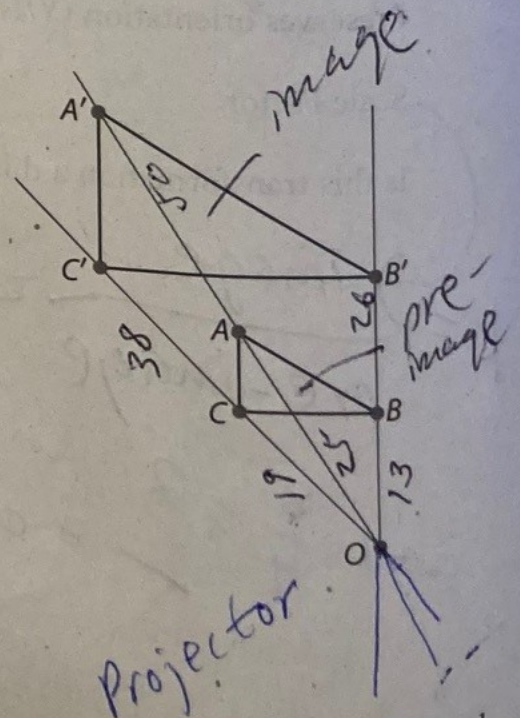
OA' = 50 mm

OB' = 26 mm

OC' = 38 mm

The scale factor is 2 to 1.

$$k = \frac{\text{image}}{\text{pre-image}} = \frac{38}{19} = 2 \text{ or } 2 \text{ to } 1 \text{ or } \frac{2}{1}$$



- B Draw $\overrightarrow{AA'}$, $\overrightarrow{BB'}$, and $\overrightarrow{CC'}$. Measure from each point to the intersection O to the nearest millimeter.

pre-image $OA = 60 \text{ mm}$

image $OA' = 30 \text{ mm}$

$OB = 38 \text{ mm}$

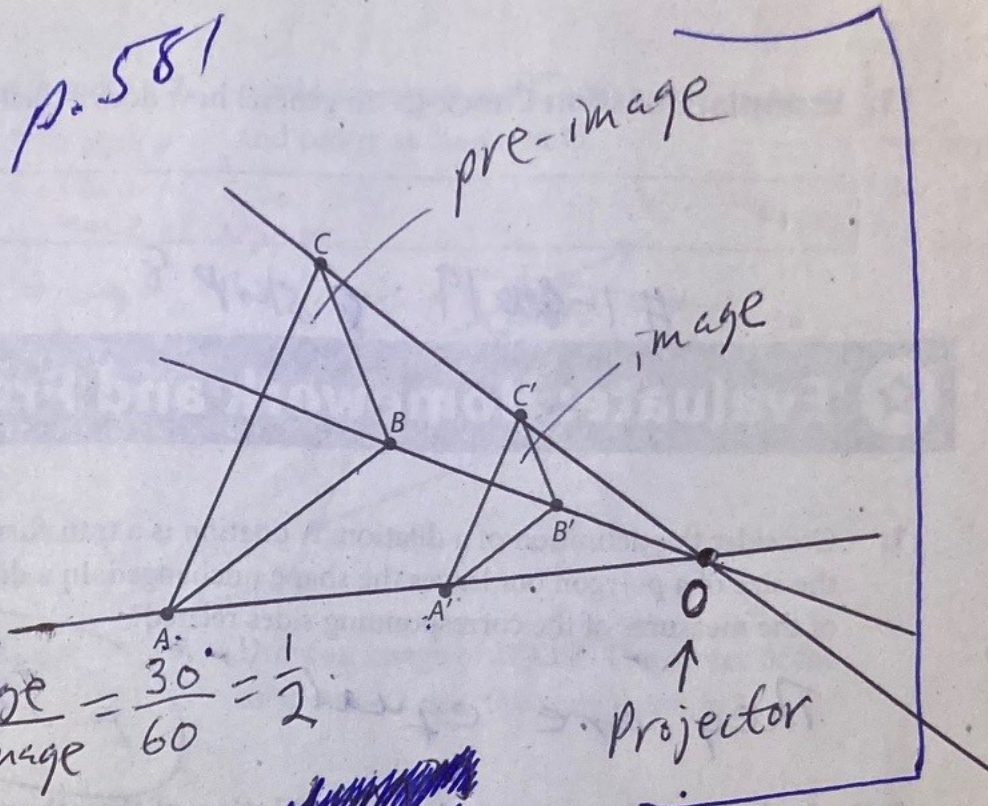
$OB' = 19 \text{ mm}$

$OC = 52 \text{ mm}$

$OC' = 26 \text{ mm}$

The scale factor is $\frac{1}{2}$

$k = \frac{\text{image}}{\text{pre-image}} = \frac{30}{60} = \frac{1}{2}$



Reflect

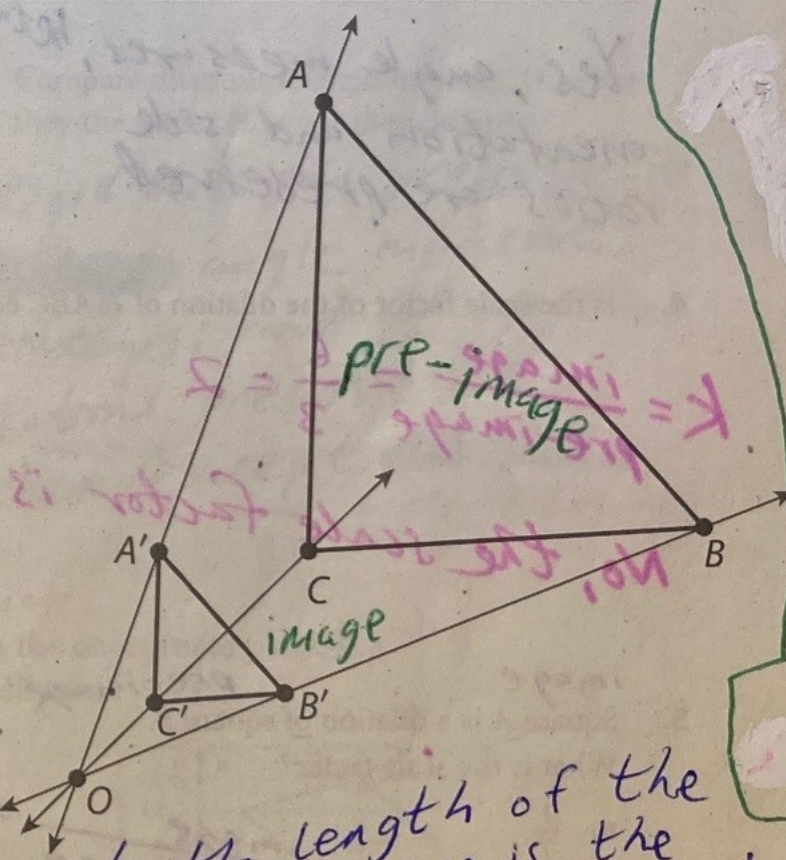
7. For the dilation in Your Turn 5, what is the center of dilation? Explain how you can tell without drawing lines.

Your Turn

8. Determine the center of dilation and the scale factor of the dilation.

$OA' = 19 \text{ mm}$ cm, $OA = 57 \text{ mm}$

The scale factor of the dilation is $\frac{1}{3}$



Elaborate

9. How is the length of the image of a line segment under a dilation related to the length of its preimage?

The ratio of the length of the image to the length of the pre-image is the scale factor, k .

10. **Discussion** What is the result of dilating a figure using a scale factor of 1? For this dilation, does the center of dilation affect the position of the image relative to the preimage? Explain.

A dilation by a scale factor of 1 will leave the figure unchanged. It will remain in the same position no matter what point is used as the center of dilation.